Math

University of Texas Bulletin
No. 2742: November 8, 1927

Physical & Applied Sci. Serials

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XII, No. 1

LIBRARY
NOV 29 1927
UNIVERSITY OF TEXAS

PUBLISHED BY
THE UNIVERSITY OF TEXAS
AUSTIN

Publications of the University of Texas

Publications Committees:

GENERAL:

FREDERIC DUNCALF
D. G. COOKE
J. L. HENDERSON
H. J. MULLER
G. W. STUMBERG
HAL C WEAVER
A. P. WINSTON

OFFICIAL:

E. J. MATHEWS
W. J. BATTLE
R. A. LAW
F. B. MARSH
C. D. SIMMONS

The University publishes bulletins four times a month, so numbered that the first two digits of the number show the year of issue, the last two the position in the yearly series. (For example, No. 2201 is the first bulletin of the year 1922.) These comprise the official publications of the University, publications on humanistic and scientific subjects, bulletins prepared by the Division of Extension, by the Bureau of Economic Geology, and other bulletins of general educational interest. With the exception of special numbers, any bulletin will be sent to a citizen of Texas free on request. All communications about University publications should be addressed to University Publications, University of Texas, Austin.

UNIVERSITY OF TEXAS PRESS, AUSTIN

University of Texas Bulletin
No. 2742: November 8, 1927

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XII, No. 1

PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH, AND ENTERED AS SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin
No. 2742: November 8, 1927

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XII, No. 1

Edited by

C. D. RICE
Associate Professor of Applied Mathematics

and

MARY E. DECHERD
Adjunct Professor of Pure Mathematics

MATHEMATICS STAFF OF THE UNIVERSITY OF TEXAS

P. M. Batchelder
H. Y. Benedict
J. W. Calhoun
C. M. Cleveland
A. E. Cooper
Mary E. Decherd
E. L. Dodd
H. J. Ettlinger
Helma L. Holmes
Goldie P. Horton
R. L. Moore
M. B. Porter
C. D. Rice
W. J. Trjitzinsky
H. S. Vandiver

This bulletin is open to the teachers of mathenatics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

CONTENTS

Secondary Schools and Secondary Teachers of Mathematics in England and France.... C. A. Noble..... 5

Mathematics of the Senior High School..... W. D. Reeve..... 16

SECONDARY SCHOOLS AND SECONDARY TEACHERS OF MATHEMATICS IN ENGLAND AND FRANCE

By C. A. Noble

University of California

During the fall of 1922 I had the interesting experience of visiting, in England and France, a few secondary schools and some institutions in which prospective teachers of mathematics are trained. I shall attempt, in a brief paper, to give a sketch of the organization of these institutions and to set out that which seemed to me characteristic.

In England, as in the United States, the secondary schools fall into two main classes, viz., the private and the public schools. Note, however, at once that the so-called "great public schools" of England, those famous, centuries-old schools like Winchester, Eton, Rugby, and Harrow, are not at all what we understand by public schools. They are privately endowed, for the most part boarding schools; they charge high tuition; and they receive, in general, no support whatever from the public treasury. The Government-supported schools, which are of relatively recent origin, derive their support partly from state grants and partly from local taxes. They also take tuition from all pupils not specially favored with the grant of free places—in other words, there are no free secondary schools.

English secondary schools carry a wide variety of designation, such as college, institute, county council school, high school, upper school, higher school, grammar school, which is confusing, at first, to an American accustomed, as he is at home, to a standardization almost as complete as that which he finds in his sewing machines, typewriters, and automobiles.

The National Board of Education in England, the head of which is a cabinet minister, exerts a control over Government schools greater than that which is wielded by our

Bureau of Education, but less than that of the Ministry of Education in France. Its control is commensurate with its function of determining the state grant of money which is allowed to those schools which are found by the board's inspectors to be "efficient." The great public schools are substantially free from control by the Board of Education and they guard their independence jealously. In case, however, they request inspection by the board they surrender automatically a measure of self control.

It is difficult to compare the organization of the English secondary school with that of the American high school. The great majority of our schools are of the four-year type. The pupil here remains normally one year in each class and leaves the school at the end of four years. It is different in the English school. It has a junior department and a senior department. There is a division into forms, but this is not the same as our division into classes. A form may connote a year, or it may not—a pupil may be in form a year, or more, or less. A pupil does not necessarily leave the English school upon graduation—there is no graduation in the sense in which we know it. There does come a moment, however, when the English boy is ready to matriculate at the university. He may then, by passing an examination, leave the school with a certificate; he may, by passing an examination—the same examination, perhaps, but with higher grade—matriculate at the university. This moment arrives normally, when the pupil is about sixteen. It may come earlier, but a boy is not admitted to the English university under sixteen. One might speak of this as the leaving age or the matriculation age.

On the other hand, the English boy may stay on at his school for one, two, or more years, going into the higher forms.

In a certain sense, however, the secondary school course in England is usually four years in duration, even if the fact is disguised by overlappings at the beginning and by superimposed post-leaving classes at the end. At the Government schools, and to an increasing extent at the great public schools, there is a core of training imposed upon all pupils,

which is designed to give a general education to those who quit school with the leaving certificate, and to prevent too early specialization by those who will go to a university.

The leaving certificate or matriculation, results from an examination set by some one of several boards, the particular board being selected by the authorities of the school. Thus there is the Oxford local examination, which corresponds to "the responsions" given at Oxford; the Cambridge local examination, which corresponds to the "previous examination" or "little go," given at Cambridge—the "local" implying that the examination takes place at the school instead of at the university. These boards are sometimes composed by grouping combinations of several of the dozen or more English universities.

The first thing that impressed me upon visiting an English school was the obvious intent to give the ambitious and capable pupil the opportunity to advance as fast as he would —he is not held down to the speed of the majority of the class, as is the case here. Classes are divided into sections, or sets, or blocks, according to ability. A certain minimum of attainment is required for the leaving certificate, but the better boys do more than this and progress as rapidly as they will.

The second thing to strike me was the fact that the pupils study mechanics in all the English schools. Professor Fawdry told me that, in his judgment, a great strength of the English secondary school lay in the fact that elementary mechanics were taught side by side with mathematics to the young pupils—that it was perfectly feasible and eminently desirable that mechanics should be taught to pupils of the age of thirteen. The examination for the leaving certificate includes an alternative paper in mechanics.

The four-year course includes algebra, geometry, trigonometry, and a little mechanics and calculus, all taught by the teachers of mathematics. The curriculum does not include substantially more mathematics than ours does, but the English pupils begin the mathematics somewhat younger and they carry several mathematical subjects abreast through several years. English teachers with whom I

talked voiced the conviction that the English plan, in this respect, was better than our plan of teaching algebra, geometry, and trigonometry tandem. On the other hand, these teachers complained to me that in their schools too many subjects are studied simultaneously (six, seven, or even eight) totaling thirty to thirty-five periods a week.

Another striking thing about the English schools is the fact that the boys headed for the university quite commonly do not matriculate at sixteen when they have passed the matriculation examination, but stay on at the school for two or more years, concentrating their attention on mathematics and mechanics. There are several reasons for this. Some boys are too poor to stand the expense of college life. Moreover, there is a feeling that the boy of sixteen or seventeen is better off, studying under the teachers he knows, than he would be in the greater freedom of the college town. The principal reason seems to be that the competition for university scholarships is so keen that a boy must study a couple of years longer if he would win one. This added study at school and consequent entrance into the university at a higher level, does not reduce the time required in residence at the university for a degree. The boy must still remain three years at Cambridge or at Oxford for the pass degree, or four years for the honors degree. The scholarship examination which admits him to college is the equivalent of the examination given in college a year after matriculation. By so entering with what we should call advanced standing he gains certain privileges, the most important being the opportunity to plunge at once into advanced work. The colleges at Cambridge and Oxford offer a large number of scholarships for promising students of mathematics—about fifty at Cambridge alone. Students enjoying these scholarships stay on an extra year for the honors degree, a degree which in the range and severity of the training demanded in mathematics bears comparison with the doctor's degree at an American university.

A noticeable feature in the class work of all the schools which I visited was the apparent absence, or infrequent use, of textbooks. The plan seemed to be that the headmaster

prepared a syllabus in mathematics for the school which all the teachers followed. This appeared to mean that mathematics was presented to the pupils, even to boys fourteen or fifteen years old, by lectures. I made numerous inquiries on this point and found that the practice varied, even in the better schools; but that the assignment of work to a class from a textbook was less general there than here.

All advancement by the pupil in an English school is by examination. There appears to be no such thing as promotion from one status to another by certification without examination, as is the case here. In fact, it sometimes, and to some people, seems as though the entire channel of education in England were directed solely to the passing of examinations. Certain it is that the necessity of passing examinations exerts a marked influence on the instruction in the secondary schools and in the character of English textbooks.

There appears to be but little provision, as yet, in England for the professional training of prospective teachers of secondary school mathematics. There are no legal requirements in force, as yet, which define the qualifications of a secondary teacher. Each university has a department of education, and there are special institutions for training. Some of these are integral parts of universities, as at Liverpool and at London; others are attached to secondary schools. Candidates for training must have a bachelor's degree or its equivalent. Those with pass degrees are, in the main, looking toward primary teaching—those with honors' degrees toward secondary teaching.

Two of the inspectors of the Board of Education, while informing me as to the present status of teacher-training in England, passed remarks which made me feel quite at home. One told me that the shortcoming of those who were engaged in training secondary teachers was that they attempted to give general training, instead of particular training in the special subject which the candidate planned to teach. Another inspector declared that the professional trainer was bent on training a candidate in methods of teaching a subject of which both the candidate and the

trainer were ignorant. I assured them that England would not take precedence over my country in these respects.

The plan now favored by the Board of Education, and which is being widely practiced, is to give the candidate a course in pedagogy and to send him for practice teaching to an approved school where counsel and guidance can be got from the teaching staff. This works well in centers of dense population, as for example in Manchester, where there is a large number of available schools. The headmaster at one of the great public schools told me that when he needed a teacher of mathematics, he sought, through an agency, an honors graduate, preferably from Cambridge. If the candidate was without experience, he was placed on probation under the guidance of the teachers of the school. Such a candidate would have had a severe course in mathematics, in amount considerably more than would be represented by our twenty-four unit major and would have had, in addition, very substantial training in mechanics.

The state schools of France constitute a well organized and closely knit system of education which embraces the primary and secondary schools, on the one hand, and the universities and other superior state schools on the other. The assemblage of Government schools (primary, secondary, and superior) in the geographical division known as an arrondissement is called an académie, presided over by a recteur. There are thus sixteen académies in France. The totality of académies constitute the université de France, at the head of which is the minister of public instruction, with the title of Grand Maitre.

Beside the state schools, called lyceés, which correspond to our primary, grammar and high school combined, there are municipal colleges and private colleges; but I shall confine my remarks on secondary schools to the lyceés.

The lyceé is a twelve-year school, of which the secondary part, the last seven years, is divided into two cycles of four and three years, respectively. The second cycle has several sections, according as the pupil elects to emphasize language, ancient or modern, or science and mathematics.

The lyceés, although state schools, are not free. They are planned to accommodate internes, or those who live in the school, as well as day pupils. Some of the latter have meals at the school. Compulsory education does not extend to the secondary grade.

The pupils at a lyceé are a select lot, the work is severe, and little mercy is shown to those who cannot keep up. The periods are an hour long. Of these a pupil has above thirty a week. I visited classes of fifteen-year-olds where the sessions for mathematics were double periods. The pupil ordinarily graduates and obtains his bachelor's degree at sixteen or seventeen, after passing a state examination. The training in the lyceé is so thorough-going that a bachelor's attainments rank him safely with a college junior in America. The University of California has repeatedly allowed the status of junior to a lyceé graduate.

In the lyceé's, as in English schools, algebra, geometry and trigonometry are taught abreast throughout the course, and the pupils also get some analytic geometry and elementary notions of calculus. Textbooks are used, prepared in accordance with the state program, but I did not see an assignment of a lesson by pages in a textbook, but rather an indication of topics, as in the English school. The teachers in both French and English schools are likely to follow a syllabus. The French boys whom I saw showed more capacity to take neat and intelligent notes than would be possible here. One of the striking things in the class work of the young boys in the lyceés which I visited was the readiness with which they talked, the speed with which they dissected problems when sent to the board, and their familiarity with mathematical tools, such as the discriminant of a quadratic, the significance of the derivative and its sign, and the use of determinants.

After having finished the twelve-year course at the lyceé and having obtained the bachelor's degree, the French boy is entitled to enter the university. But if he is interested and proficient in mathematics he does not do this. The work at the university is keyed at too high a pitch. On the contrary, he returns to the lyceé from which he graduated, or

enters one of the great lyceés of Paris, and studies for at least two years more in one of the special classes in mathematics designed for those who aim to prepare for a state career. From this time on his energies are bent in the direction of intense specialization in mathematics and science. The course is very exacting. Of the thirty to thirty-five hours of class work per week, twelve to fifteen are in mathematics alone. The work is entirely by lectures and no mercy is shown to the pupils. It is an inexorable sifting process. As several of the teachers told me, in classes which I visited, the school was not at all concerned for the pupils who fell behind.

Secondary schools in France are not all for boys any more than they are in England. I visited a girls' school in Paris as well as in London. Theoretically they were of the same standard as those for the boys. I thought I saw a difference, a difference comparable to that which obtains between the church schools and the public high schools in California. The pupils were industrious and very amenable—docile. They were apt to respond in chorus to questions, were apparently disposed and allowed to do more committing to memory than were the boys, and seemed, in reciting, to speak less from conviction and more to satisfy the questioner than was the case with the boys.

The teachers of mathematics in the French lyceés are, without doubt, the most select and highly trained secondary teachers in the world. The training to which they are subjected is identical with that which is required of professors of mathematics at the French universities. In both cases the diploma *licence ès science* is necessary, and usually the competitive examination called *agregation* is passed. Let us follow, briefly, the course of preparation of a future teacher. We have just seen him returning to his lyceé, after obtaining his bachelor's degree, and entering the special class in mathematics for a two years' grind. After this period of intensive special study he takes a competitive examination for admission to the great state normal school, the Ecole Normale Superieure. This examination is set by the Government and the number received into the Ecole is

determined by the capacity of the school. In 1908, 274 boys tried the examination; 22, or 8 per cent, were admitted. Those who fail in this examination return to the lyceé for another year of study and try the examination again. Upon this second trial not more than 25 per cent to 30 per cent succeed. After a third trial at which not more than one-half pass, the applicants reach an age limit and those who have failed betake themselves into channels where the demands are less exacting.

The future lyceé or university professor is now admitted to the Ecole Normale, where he remains three years. Formerly all the students at this school were internes, but recently externes have been admitted. The student's work here is largely in pedagogy. Simultaneously, however, he hears lectures at the Sorbonne and at the College de France. His main purpose, during this time, is to prepare for two examinations—the *licence* and the *agrégation*.

The diploma licence és science is necessary for all those who take up secondary teaching. It implies three "certificates," one for each of three groups of subjects. For the teacher of mathematics these groups are: (1) Differential and Integral Calculus, (2) Rational Mechanics, (3) General Physics or some advanced topic in mathematics. The examinations are offered twice yearly. Each examination, successfully passed, entitles the student to a certificate in that subject. The examination in a subject consists of three parts—the *épreuve écrite*, *épreuve pratique*, and the *épreuve oral*. The first two are written and about four hours long each; the oral test lasts fifteen to twenty minutes. The scope of these examinations is indicated by the fact that the course in calculus on which the first examination is based is a series of lectures at the Sorbonne covering the ground of Goursat's Cours d' Analyse.

Unlike the *baccalaureat* and the *licence*, the *agregation* is a competitive examination. The number who become agrégés each year is fixed in advance by the minister of public instruction in accordance with the current need of teachers in the lyceés of the state. In recent years the

number has been about fourteen and the number of candidates about eighty. The candidates for this examination must have four certificates: (1) Differential and Integral Calculus, (2) Rational Mechanics, (3) General Physics, (4) a subject chosen in advanced mathematics.

In preparation for the agregation the student at the Ecole Normale disposes his time as follows: During each of the first two years he passes examinations for two of the four requisite certificates. During the third year he prepares for the agrégation proper. This examination is unique in its difficulty and exactions. As it is planned for selecting the most efficient young men of the country for teaching the mathematics in the lyceés, the examination turns largely on the subjects taught in these schools. It consists of *épreuves préparatoires* and *épreuves definitives*. The former are four written examinations, each of seven consecutive hours duration (7A.M—2P.M.). The first two of these are on subjects chosen from the program of the lyceé, the last two are based on the work done by the candidate at the university. The épreuves préparatoires are held at the seats of the various academies of France. The candidates who attain a sufficiently high standard are declared "admissible." They must then go to Paris for the épreuves definitives. These consist of two written examinations, usually four hours long, and two leçons, or specimens of classroom instruction. The subject for the leçon are drawn by lot, and for each leçon the candidate has four hours to think over what he is going to say.

The professional training at the Ecole Normale includes, besides courses in pedagogy, a course in elementary mathematics from an advanced standpoint. I attended several sessions of this course. The students were required to lecture for an hour on some topic of the lyceé curriculum, after which their presentations were criticized as to content and method. This is quite like what is attempted here with our candidates for the teachers' certificate. The students at the Ecole Normale have three or four weeks of practice teaching at a lyceé.

At Sevre there is a normal school for the training of women teachers, with the same standard, theoretically, as that of the Ecole Normale, to which a school for practice teaching is attached.

I shall try to indicate my impression of the scheme of education as I have seen it operating in France, England, and the United States. In France the state is little or not at all interested in universal secondary or higher education. Its concern is to obtain for its own service the cream of its intellectual power. Its educational system, therefore, in its upper reaches, is designed as a sifting process to the end that at the top there shall emerge the most gifted individuals—highly and specially trained, in number just adequate for the current needs of the state. In England there is deeper concern for general education than in France, but it is still within bounds. General education is not free. A charge is put upon every aspirant who is not specially relieved. Relief is based upon examination as to fitness. The plan of instruction in the English system provides for the certification of those who reach a certain minimum of attainment; but it provides also a flexible arrangement which permits a student to pass beyond that minimum and encourages the gifted student to progress as fast and as far as he will. In the United States the growing tendency is to offer to every boy and girl, free of charge, and regardless of state expense, the opportunity to obtain a grammar school, a high school, a college education. This has already landed us in "quantity production" in our educational institutions quite as marked as in our automobile factories. It means the dilution and waste of the energy of teaching bodies and with, as yet, too little opportunity for the capable student to come into his own.

THE MATHEMATICS OF THE SENIOR HIGH SCHOOL*

By W. D. Reeve

Associate Professor of Mathematics, Teachers College

PRESENT STATE OF AFFAIRS

Mathematics *for Everybody.* The typical junior high school constituting grades seven, eight, and nine, now offers some kind of a general introductory course in mathematics. The content, methods of teaching, and order of presentation of topics may vary from place to place, but the great central purpose is fairly uniform. This type of course should prevail in all seventh, eighth, and ninth grades throughout the country even though the junior high-school organization as such is not universally recognized. Everybody who wishes to be educated should know something of the essential applications of arithmetic, of the central features of intuitive geometry, of the meaning and uses of algebra, of the nature and value of numerical trigonometry, and of the meaning of a demonstration in geometry. The course described above should be required of all pupils whether they leave school at the end of the ninth grade or go on to the senior high school. In four-year high schools it will be necessary to require some kind of course in the ninth grade that will give the pupils a training that, in the long run, will represent what the educated citizen should know about mathematics as a tool.

Purpose of Senior High School Mathematics.—The purpose of the mathematics course in the senior high school is to meet the needs of the following four groups of pupils:

1. Those who intend to go on to colleges and techinal schools.
2. Those who are going to specialize in commercial work that requires mathematics, especially algebra.
3. Those who expect to specialize in science.

---

*From *Teachers College Record.*

4. Those who desire to study mathematics further because they like it.

Because of the purpose outlined above it would seem that the senior high-school course in mathematics should be made elective.

Content of the Course.—Obviously the content of any course in mathematics should be determined by the purpose in mind. At the present time there is little attempt in most high schools to consider the needs of individual groups like those above, or even of individuals themselves. Indeed, this may continue to be true of the majority of the small high schools of the country.

Geometry is required of nearly everybody, regardless of whether the pupils taking it can all profit by its study. This condition is due almost entirely to the dominance of extra-mural examination boards who have in mind only the needs of the first group mentioned above. Plane geometry is taught in all schools,—usually in the tenth grade. But beyond that point there is no uniform practice. In some schools intermediate algebra and solid geometry are taught in the eleventh year, in others these subjects may be deferred until the twelfth year. In a few high schools trigonometry and college algebra are offered in either the eleventh or twelfth year, but the number of such schools is very small. Even where such higher courses are offered, the situation results not so much from a desire to meet the needs of any particular group as it does because of the influence of some local teachers. In rare cases we have schools in which analytic geometry and calculus are offered, as is the case in Wadleigh High School in New York City, the University High School, at Oakland, Cal.,[1] and in certain other laboratory or experimental schools like the Horace Mann School and the Lincoln School at Teachers College. The report of the recent National Committee on the Reor-

---

[1]Durst, Ethel H. "Calculus for High Schools." *University High School Journal*, June, 1926, pp. 135–167.

ganization of Mathematics gives a list of such schools and also a statement of the work they are trying to do.[2]

We may not be sure what the new course in mathematics ought to be, but we know that the present situation in most schools is very unsatisfactory.

PROBABLE FOUNDATION COURSES

Elementary Algebra.—In the absence of a junior high-school organization many four-year high schools will continue to offer, as they do now, a course in elementary algebra. No course in the high-school curriculum has made more progress recently than algebra. The recent changes in the spirit of teaching and in the purpose of education make it necessary for textbook writers to set forth the results of the latest investigations in the field of algebra in a manner which shall meet the needs of the classroom. There are four topics of algebra that the educated citizen needs to know, whatever his occupation may be. They are the formula, the simplest types of equation, the graph, and directed numbers. These topics are important because they contain the information which the citizen should possess for his everyday reading in current literature, in popular science, in the newspapers and other periodicals of the home, or in the simpler class of technical handbooks. These topics, therefore, are fundamental, either for the pupil who is to go on with more advanced work or for the one who, upon leaving school, is to take up at once some line of business or a home career.

Plane and Solid Geometry.—The purpose of the geometry course is to show the pupil the meaning of a demonstration, to enable him to have the satisfaction of proving something, to let him see the significance of mathematical precision, and feel the joy that comes from discovering absolute truth. In other words, the work in geometry is the only course in

---

[2] *The Reorganization of Mathematics in Secondary Education.* Report of the National Committee on Mathematical Requirements. Chap. XII.

deductive logic which a pupil can get in a secondary school.[3] It is better than pure logic because it has in it figures that illustrate the things that are said and done.[4] If geometry is not taught to give a training in deductive logic, in the power to prove one's own statements, or if it does not do both, then it is not worth teaching.

In view of the pressure of time and the needs of the curriculum, it is clear that we can no longer justify in all schools, separate courses in plane and solid geometry extending over a year and a half. The tendency in many schools at the present time is to devote one year to the study of both and to combine them properly for teaching purposes.

The difficulty with the ordinary geometry course in American high schools is that we have held too strongly to the systematizing motive and have given the pupils an unadulterated form of Euclid. We should profit by the suggestions of The Mathematical Association[5] even though we need not imitate in every detail the practice which its report advocates. The British list is not so long as ours used to be, and it spreads the work out over a longer period of time so that it is better understood. Our course must necessarily be somewhat uniform on account of the mobile character of our population, but it need not be so extensive as formerly.

Number and Importance of Propositions.—The results of geometry teaching can be improved by decreasing the number of theorems we teach, by concentrating on those propositions which are unquestionably fundamental, by recognizing the importance of original work, and by emphasizing ability to demonstrate rather than the number of propositions studied and perhaps often memorized. We now expect the pupil to master only about one hundred

---

[3]Carson, G. St. L. *Mathematical Education.* Pp. 85–112. Ginn & Co., 1913.

[4]Smith, D. E., *The Teaching of Geometry.* Ginn & Co., 1911.

[5]*The Teaching of Geometry in Schools.* Mathematical Association. G. Bell & Sons (London), 1923.

propositions.[6] One modern text has nearly one thousand original exercises.

We ought to take many theorems for granted. The three on congruence in the early part of the work in geometry might well be postulated if it were not for extramural examinations. Some teachers say that plane geometry cannot be taught in one year. This presupposes a "definite amount of ground" to be covered, which is a myth. We cannot teach all that is known of geometry in many years or even a lifetime. We must remember that for some pupils a few propositions will suffice while for others the number must be much greater.

Plane and Spherical Triginometry.—Trigonometry is strictly a high-school subject. Pupils generally like the subject when it is well taught and we have evidence to show that they do as well with it as do college freshmen two years later.

In a study carried on by the writer at the University of Minnesota High School, a group of high-school juniors made, on the same series of examinations in trigonometry, a semester average 6 per cent higher than a freshman class in the University studying the same subject. This does not prove that the high-school pupils were better, but it certainly shows that the subject was not too difficult for them.

The fundamental ideas of both plane and spherical trigonometry can easily be covered in a half year's time and for many pupils it need occupy no more than twelve weeks. Many pupils can learn it in much less time. Since, however, the applications of spherical trigonometry are almost entirely confined to astronomy and geodesy, it has limited sanction under our present conditions.

Intermediate Algebra.—For those who intend to proceed further in science, economics, mathematics, or other technical subjects, either in college or elsewhere, more mathematical work is necessary. The course in intermediate algebra still occupies a large place in many schools and

---

[6] *Revised Report on the Requirements in Plane Geometry.* Pp. 9–17. College Entrance Examination Board, 1923.

should continue to do so. In some schools it is amplified so as to occupy a year's time; but a half year should be sufficient, especially if needless reviews of the elementary course are omitted and the remainder of the course is modernized. There is the same need now for reform in the content of some of our intermediate algebra courses as there was ten years ago in elementary algebra.

The essence of the course in algebra should be functional relationship,—the notion of the dependence of one quantity upon another. In some schools trigonometry, analytic geometry, and the calculus are introduced at this point to help bring this about. Professor Nunn, the well known English writer, says, "As soon as the symbols of trigonometrical ratios are recognized as capable of entering into formulas and of being manipulated, they should be regarded as belonging to the vocabulary of algebra. There is, in deed, no principle except the invalid principle of formal segregation, upon which we can include the study of x or a in the algebra course and exclude sin x or tan x. All alike are pieces of symbolism invented for the description and interpretation of facts of the external world. Each represents a typical kind of function. To each corresponds a specific form of curve which may be regarded as the graphic symbol of the function. Both algebra and trigonometry would gain by fusion,—the former through an added variety and richness in the illustration of its main themes, the latter by the removal of the excessive formalism which at present obscures its value and interest for the beginner."

PROBABLE ALTERNATIVE COURSES

College Algebra.—In an increasing number of schools there is offered in the senior year a half-year course in college algebra. This would be one profitable way to spend the time if such a course included what is essentially college algebra. In too many places, however, such a course is college algebra only in name, being mostly reviews of the more difficult parts of elementary and intermediate algebra

with only an occasional glimpse at new material on the college level. One solution for such a situation is the general adoption of the general mathematics program all through the senior high school,—a plan which will presently be discussed.

Courses for Future Mathematicians.—For the future mathematicians there will probably be offered certain half-year courses that will be largely informational and basic. Chief among these are college algebra, analytic geometry, projective geometry, the calculus, and possibly a course in what we might call college geometry,—geometry to correspond with the course called college algebra.

The decision as to which of these courses is to be offered will doubtless be left to the particular wishes of the instructor who will have to teach the subject.

Courses for Vocations.—There will also be offered in some of our senior high schools courses for those who are preparing to enter certain vocations. Thus we may find certain schools offering a course in statistics for those who may later need such work, a course in mechanics for those so inclined, a course in commercial algebra or in commercial arithmetic, and possibly a course in machine-shop mathematics.

General Mathematics.—There is a demand in an increasing number of four-year high schools for at least a two-year course in general mathematics for those pupils who remain until they finish the tenth grade. For those who finish their high-school educations and who desire to study mathematics further, the general mathematics program should be extended to cover the entire four years.

The demand for such a course is due to the wide differences that exist today both in the work of the junior high school and in the purposes of instruction in the senior high school. Thus, we find one group of pupils in the four-year high school whose previous work in mathematics has been limited to arithmetic, another whose course may have included general mathematics like that set forth in the first or second book of a modern junior high-school series, still another group which may be preparing to go to college or

technical schools, and a few others who intend to go into business or enter upon some other career. It would be impossible for the traditional courses of study to meet the needs of all such groups.

Work of the Ninth Grade.—Experience has shown that the most satisfactory division of emphasis in a general mathematics course for the ninth grade of a four-year high school leads to making the work center around algebra, so as to give the pupil an understanding of what the subject means and how to use it. The algebra should be combined wherever possible with numerical computation, with the facts of geometry that are included in what has come to be called intuitive geometry, and with the elementary principles of numerical trigonometry.

The work in numerical computation keeps the pupil conscious of the fundamental importance of arithmetic in all mathematics work; the work in intuitive geometry offers the best possible preparation for a unit of demonstrative geometry during the last part of grade nine, or for the geometry work of the tenth grade; and the work in trigonometry, which is now being generally included as part of ninth grade algebra, gives the pupil an excellent idea of indirect measurement.

Work of the Tenth Grade.—The central feature of this year is the fundamental elements of plane and solid geometry. If a pupil has had the general mathematics course of the ninth grade described above, he is familiar with a large number of definitions and concepts that will form an excellent basis for the work of the tenth grade. Algebra and trigonometry should be used wherever they help to clarify the proofs. Trigonometry correlates well with intuitive geometry and algebra, but not with demonstrative geometry. Just to the extent to which it is permissible to introduce mensuration, just so far is it feasible to introduce trigonometry into demonstrative geometry. Both mensuration and trigonometry may have a place at this time for the purpose of changing the emphasis and of providing the pupils with a mental rest from the pursuit of logical demonstration.

Work of the Eleventh Grade.—For those who continue the study of general mathematics through the eleventh grade the course should be built up around algebra, trigonometry, and the simpler elements of analytic geometry as the central features. Some teachers may prefer to introduce the calculus by this time or even earlier. And there is no reason why this should not be done in certain schools. The following outline for the work of this grade is meant to be merely suggestive of the general field and not of the order of presentation:

I. Dependence.
   1. Meaning of dependence,—functional relationship.
   2. Review and extension of the earlier work on algebraic functions.
   3. Trigonometric functions as examples of transcendental functions.
   4. Functional notation.
   5. Evaluation of functions.
   6. Determination of functions.
      a. From empirical data and tables.
      b. Empirical *vs.* arbitrary functions.
   7. Classification of algebraic functions.
   8. Graphs of algebraic functions.
      a. Variation of function.
      b. Graphical introduction to maxima and minima.
   9. Fundamentals of statistical method.
      a. Theory of measurement.
      b. Fundamental ideas.
      c. Applications.

II. Trigonometric Functions of Any Angle.
   1. Review and extension of the earlier work.
      a. Ratio definitions, including secant, cosecant, and cotangent.
      b. The right triangle where the acute angle is 30°, 45°, or 60°.
      c. Similarity of triangles. No trigonometric function of a given acute angle has more than one value.
      d. Given the value of a trigonometric function of an acute A, to construct A and determine the other trigonometric functions.
   2. Changes in the functions of A as A increases from 0° to 360°. Graphical introduction.

3. Positive and negative angles of any size.
   a. Angles whose initial sides and radius vectors are respectively identical.
   b. The signs (quality) of the functions of angles in the various quadrants.
4. Functions of $(-A)$ in terms of A.
5. Functions of $n\times 90^\circ \pm A$, $n\times 180^\circ \pm A$, and $n\times 360^\circ \pm A$.
6. Line definitions of the trigonometric functions.
   a. Graphical representation.
   b. Application to wave motion, electricity, and the like.
7. Table of natural functions.
8. Applied problems.
9. Fundamental trigonometric relations like

$$\sin x = \frac{1}{\csc x} \text{ and } \tan x = \frac{\sin x}{\cos x}.$$

   a. Proving identities.
   b. Applications to physics, navigation, and surveying.

III. Logarithms and Applications.

IV. Solution of Triangles.
   1. Right trangles.
   2. Oblique triangles.
   3. Applications.

V. Identities.
   1. In elementary operations.
      a. Work on functions extended.
      b. Use of analytic method, especially in proportion.
      c. Factoring.
         (1) Review and extension of earlier types.
         (2) Factor theorem.
         (3) Factors of $x^n \pm y^n$.
         (4) Mathematical induction.
   2. In formulas.
      a. Review and extension of earlier work.
      b. Addition and subtraction theorems of trigonometry.
      c. Trigonometric functions of $2A$ and $\frac{A}{2}$.
      d. Applications of *b* and *c* above.
         (1) In evaluating certain functions.
            For example, $\sin 75^\circ = \sin (45^\circ + 30^\circ) = ?$
         (2) In proving certain identities.

VI. Straight-Line Formulas.
   1. Distance formula.
   2. Mid point formula.

3. Slope of a line, parallel lines, a perpendicular.
4. Point slope form.
5. Slope intercept form.
6. Two point form.
7. Two-intercept form.
8. Distance from a point to a line.

VII. Equations.
1. Linear.
2. Quadratic.
a. In one unknown.
(1) Maxima and minima.
(2) Radical equations.
b. In two unknowns.
c. Theory of quadratic equations.
3. Trigonometric.
a. Radian measure.
b. Two acute angles are equal if any trigonometric function of the one is equal to the same function of the other. Explanation of how the value of an angle is obtained and thus how to solve a trigonometric equation.
c. Solution of trigonometric equations.

VIII. Series.
1. As an example of an identity.
2. General nature.
3. Classes to be considered.
a. Binomial Theorem.
b. Arithmetic.
c. Geometric.
4. Applications.

IX. General Number System.
1. Integers.
2. Fractions.
3. Negative.
4. Surd.
5. Simple imaginaries.

Work of the Twelfth Grade.—There is no reason why pupils who wish to elect general mathematics in the twelfth grade should not be provided with a suitable course in the high school. In a four-year high school, in which the writer taught mathematics, slightly over 25 per cent of several entering freshman classes elected four straight years of general mathematics. There is probably no best course and a great deal of experimental investigation should be

carried on, but the writer offers the following outline as a suggestion for such a course:

I. Variation of Functions.
  1. Graphical representation of functions, to bring out variation of different kinds.
    a. Mechanical graphs like Weather Bureau records.
    b. Review of details in graphic work, scales, and the like.
  2. Mathematical method of studying functions.
    a. Who makes such study? Illustration.
    b. Why such study is not made of all existing functions.
  3. Review of functional notation.
  4. Directed lines. Rectangular coördinates.
    a. Intercepts.
    b. Infinitely large or infinitely small functions.
  5. What is meant by a rate.
    a. Rate of change of a function.
    b. Uniform rate of change of a linear function.
      (1) Slope of a straight line $y=mx+b$.
      (2) Uniform acceleration.
      (3) Applications.
    c. Average rate of change of a function.
    d. Instantaneous rate of change of a function.
    e. Graphic methods involving *a*, *b*, *c*, and *d*.
    f. Small intervals.
    g. Interpolation by proportional parts.
  6. Important problems.
    a. Mean-value.
      (1) Average value throughout a certain interval.
      (2) Applications.
        (a) To find the distance traveled.
        (b) In physics.
        (c) In geometry.
    b. Extreme-value.
      (1) Review and extension of maxima and minima.
      (2) Applications.
    c. Zero-value.
      (1) Review of earlier equation work with emphasis on the idea of variation.
      (2) Review of graphic methods of solving simple equations and extension to the solution of higher equations.
      (3) Theory of equations.
        (a) Number of roots.
        (b) Location principle.
  7. Deriving formulas.

II. The Notion of a Limit.
  1. Elementary notion.
    a. Numerical.
    b. Geometric.
  2. Tied up with instantaneous velocity.
  3. Instantaneous direction.
    a. What a tangent line is.
    b. Slope.
  4. Explanation of limit.
  5. Explanation of notation used.

III. Differentiation and Its Applications.
  1. The idea of a derivative of a function.
  2. What $\frac{dy}{dx}$ means.
  3. Differentiation of simple functions, formulas.
  4. Maxima and minima.

IV. Integration and Its Applications.
  1. Integration explained.
  2. Formula for integration.
  3. Notation.
  4. Applications to areas, surface, volumes.
  5. Other applications.

V. Commercial Algebra.
  1. Review.
    a. Exponential and logarithmic functions.
    b. Growth curves.
    c. Compound interest.
    d. Annuities.
    e. Business depreciation.
    f. Differentiation and integration.
  2. Series.
    a. Evaluation of functions.
    b. Mathematics of investment.
      (1) Accumulation.
      (2) Life Insurance.

VI. Permutations, Combinations, and Probabilities.
  1. Permutations.
  2. Combinations.
  3. Chance explained.
  4. Simple and compound probability.
  5. Normal distributions, surfaces and curves.
  6. Errors of measurement.
  7. Applications.
    a. Life insurance problems.
    b. Mendelian theory of inheritance.

VII. The Number System Extended.
1. Review and extension of the real number system.
2. Similarly for imaginary numbers.
3. What a complex number is.
4. Polar form of a complex number. Polar coördinates.
5. Applications.